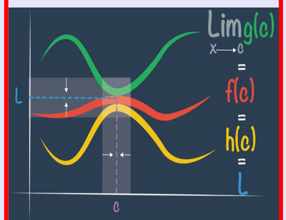
Math 261

Spring 2022

Lecture 28

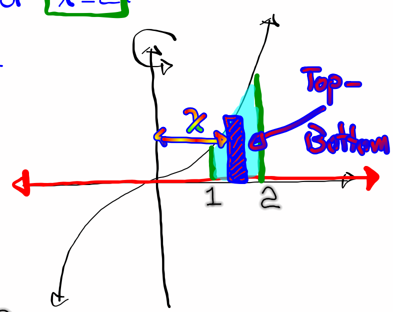


Draw the enclosed region bounded by

$y = x^3$, $y = 0$, $\boxed{x = 1}$, and $\boxed{x = 2}$

Rotate this region about the Y-axis.

Find the Volume.



Cross-Section is Parallel to A.O.R.

⇒ Shells Method

$$\int_1^2 2\pi \,(\text{distance from A.O.R.})\,(\text{Height of dx Cross-Section})$$

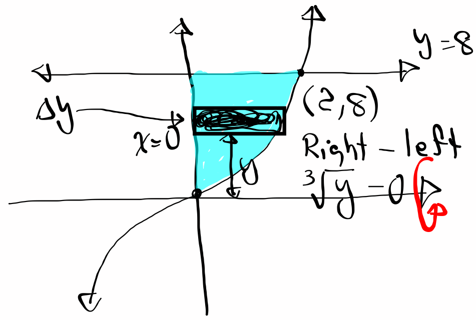
$$V = 2\pi \int_1^2 x \cdot x^3 \, dx = 2\pi \cdot \int_1^2 x^4 \, dx$$

$$= 2\pi \cdot \frac{x^5}{5} \Big|_1^2 = \boxed{}$$

Draw the enclosed region by $y=x^3$, $y=8$, and $x=0$.

Rotate this region by the x-axis.

Find the Volume.



If cross-section is vertical

washer Method

$R=8$ $\quad$ $r=x^3$

$$\int_0^2 \pi\left[R^2-r^2\right]dx = \boxed{}$$

If cross-section is horizontal

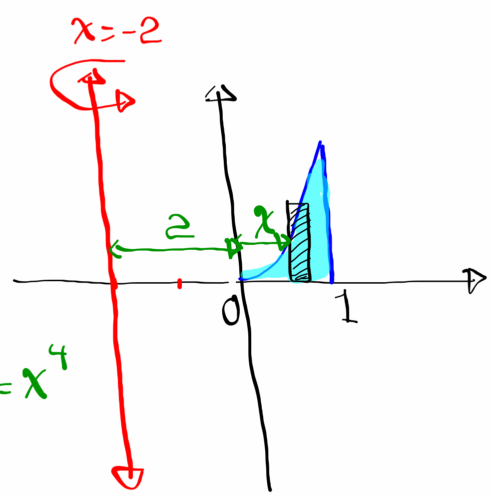
Shells Method

$$\int_0^8 2\pi\cdot y\cdot \sqrt[3]{y}\; dy = \boxed{}$$

Same Answer

Rotate the enclosed region by $y=x^4$, $y=0$, and $x=1$ by $x=-2$.

Cross-Section is parallel to A.O.R.



Distance from A.O.R. $\Rightarrow x+2$

Height of cross-section $\Rightarrow x^4-0=x^4$

By Shells Method:

$$V=\int_0^1 2\pi\cdot(x+2)\cdot x^4\,dx = \boxed{}$$

Draw the enclosed region by $y = 4x - x^2$ and $y = 3$.

$y = 4x - x^2$

Parabola

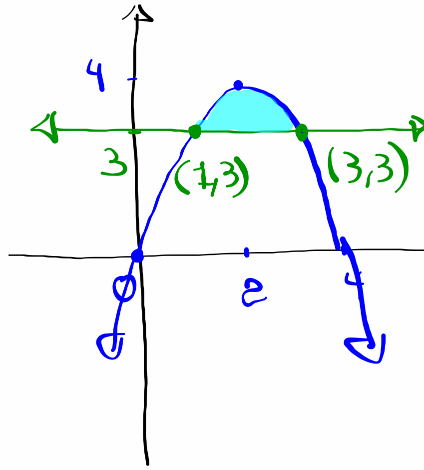
open downward

Vertex $\to y' = 0$

$y' = 4 - 2x \to y' = 0$

$x = 2$

$y = 4(2) - 2^2 = 4$



$4x - x^2 = 3$

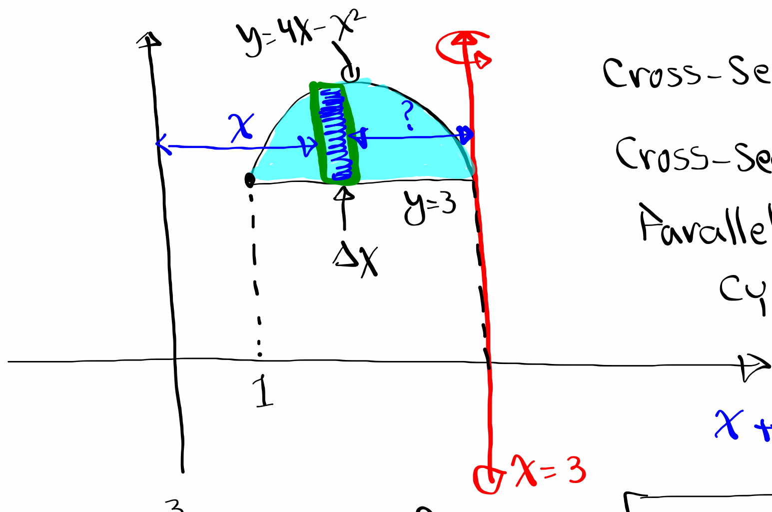
$-x^2 + 4x - 3 = 0$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1 \quad x = 3$

Rotate this region by $x=3$, and find its volume.



Cross-Section is Vertical

Cross-Section is Parallel to the A.O.R.

Cylindrical Shells Method.

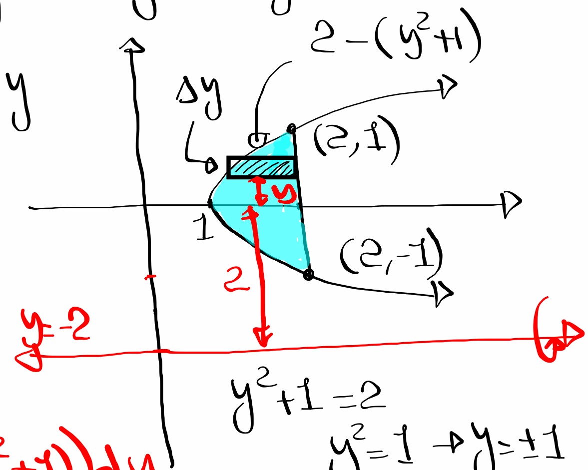
$x + ? = 3 \Rightarrow ? = 3 - x$

$$V = \int_1^3 2\pi \cdot (3-x) \cdot [4x - x^2 - 3]\, dx = \boxed{}$$

Draw the enclosed region by $x = y^2 + 1$ and $x = 2$.

Rotate this region by

$y = -2$



Find the volume

$y^2 + 1 = 2$

$y^2 = 1 \to y = \pm 1$

$$V = \int_{-1}^{1} 2\pi(2+y)\cdot\left(2-(y^2+1)\right)dy$$

$= \boxed{}$

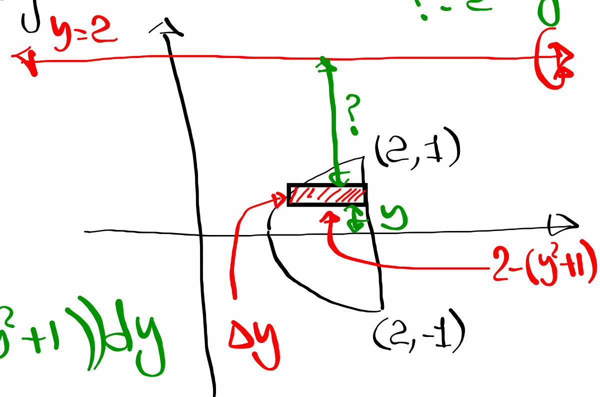
Draw the enclosed region by $x = y^2 + 1$ and $x = 2$.

Rotate this region by

$y = 2$

$y + ? = 2$

$? = 2 - y$



Find the volume

$$V = \int_{-1}^{1} 2\pi(2-y)\left(2-(y^2+1)\right)dy$$

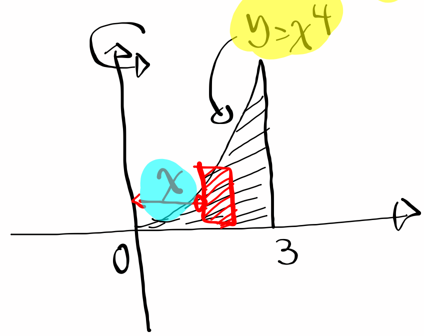
$= \boxed{}$

The integral below represents the volume of a Solid. Describe the Solid.

$$\int_0^3 2\pi x^5\,dx$$

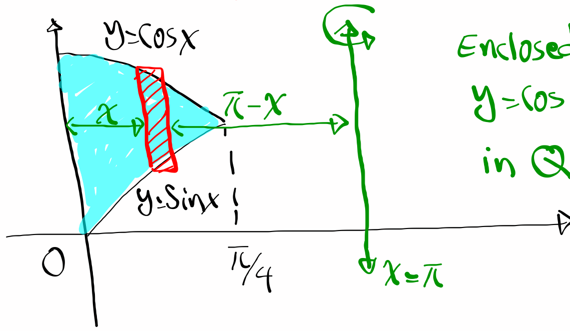
$$= \int_0^3 2\pi \cdot x \cdot x^4\,dx$$

Hint: $x^5 = x \cdot x^4$



The integral below represents the volume of a Solid. Describe the Solid.

$$\int_0^{\frac{\pi}{4}} 2\pi\underbrace{(\pi-x)}_{\substack{\text{distance}\\ \text{from A.O.R.}}}\cdot\underbrace{(\overset{\text{Top}}{\cos x}-\overset{\text{Bottom}}{\sin x})}_{\substack{\text{height of}\\ \text{the cross-section}}}dx$$

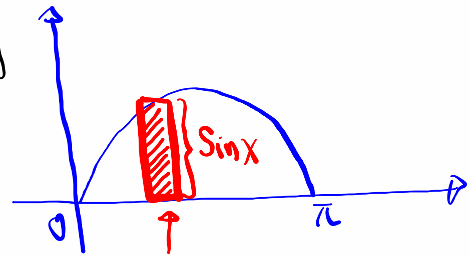


Enclosed region is by $y=\cos x$, $y=\sin x$, $x=0$ in Q I.

A.O.R. $x=\pi$

Enclosed the region by $y = \sin x$, $y = 0$ in QI.

Find the volume by rotating this region by



a) Y-axis

Shells

$$V = \int_0^{\pi} 2\pi \cdot x \cdot \sin x \, dx$$

Hint: $\int x \sin x \, dx = \sin x - x\cos x + C$

Hint: $\sin^2 x = \frac{1 - \cos 2x}{2}$

b) x-axis

Disk $\quad V = \int_0^{\pi} \pi \cdot \sin^2 x \, dx$

Hint: $\int \sin^2 x \, dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$

The integral below represents the volume of a Solid. Describe the Solid, then find the volume

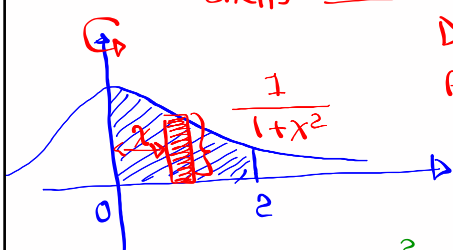
$$\int_0^2 \frac{2\pi x}{1+x^2} dx = \int_0^2 2\pi \cdot x \cdot \frac{1}{1+x^2} dx$$

height of Cross-Section

$f(x)$

Shells

Distance from Cross-Section to A.O.R.



Region enclosed by

$y = 0$, $y = \frac{1}{1+x^2}$, $x = 0$, $x = 2$

A.O.R. Y-axis

$$V = \int_0^2 \frac{2\pi x}{1+x^2} dx$$

$u = 1 + x^2$

$du = 2x\,dx$

$x = 0 \rightarrow u = 1$

$x = 2 \rightarrow u = 5$

Hint: $\int \frac{1}{u} du = \ln|u| + C$

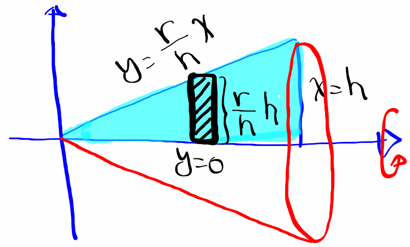
$$= \pi \int_1^5 \frac{1}{u} du = \pi \cdot \ln u \Big|_1^5 = \pi(\ln 5 - \cancel{\ln 1}^{0}) = \boxed{\pi \ln 5}$$

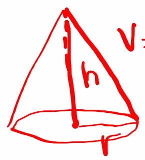
Assume $r>0$, $h>0$,

Draw the region bounded by $y=\frac{r}{h}x$, $y=0$, and $x=h$.

Rotate this region by the x-axis.

Find its volume.





$V=\frac{\pi r^2 h}{3}$

Disk

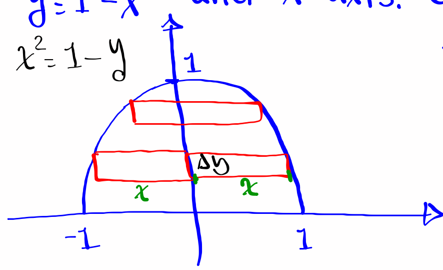
$$V=\int_0^h \pi\left(\frac{r}{h}x\right)^2 dx$$

$$=\pi\cdot\frac{r^2}{h^2}\int_0^h x^2 dx=\pi\cdot\frac{r^2}{h^2}\cdot\frac{x^3}{3}\Big|_0^h$$

$$=\pi\cdot\frac{r^2}{h^2}\cdot\frac{h^3}{3}=\boxed{\frac{\pi r^2 h}{3}}$$

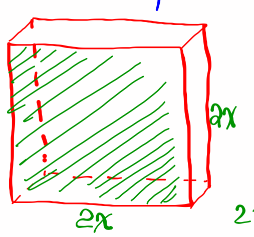
The base of a Solid is the region enclosed by $y=1-x^2$ and x-axis. Cross-Sections are $\perp$ to y-axis and are squares.

$x^2=1-y$

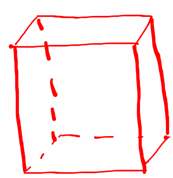


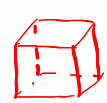
$$V=\int_0^1 \boxed{\text{Area of}}\,dy$$
Cross-Section

$$V=\int_0^1 4x^2\,dy$$



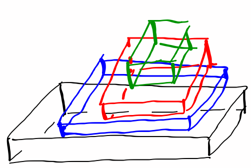
$2x\cdot 2x=4x^2$





$$V=\int_0^1 4(1-y)\,dy$$

$$=\boxed{}$$



The base of a Solid is an ellipse given by

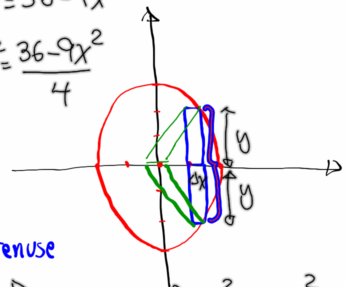
$9x^2+4y^2=36.$ $\longrightarrow 4y^2=36-9x^2$

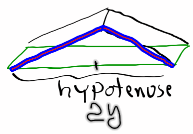
$y^2=\frac{36-9x^2}{4}$

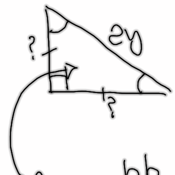
$\frac{x^2}{4}+\frac{y^2}{9}=1$

Cross-Sections ⊥ x-axis

Cross-Sections are isosceles right Triangle with hypotenuse in the base







$?^2+?^2=(2y)^2$

$2\cdot ?^2=4y^2$

$?^2=2y^2$

$\text{Area}=\frac{bh}{2}=\frac{?\cdot ?}{2}=\frac{?^2}{2}=\frac{2y^2}{2}$

$\text{Area}=y^2$

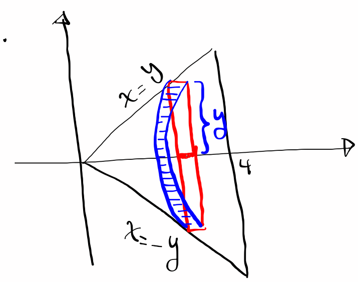
$$V=\int_{-2}^{2}\boxed{\text{Area of}}\,dx=\int_{-2}^{2}y^2\,dx=\int_{-2}^{2}\frac{36-9x^2}{4}\,dx=2\int_{0}^{2}\frac{36-9x^2}{4}\,dx$$
$\quad$ Cross-Section

$=\boxed{}$

The base of a Solid is the region enclosed by $x=|y|$ and $x=4$.

Cross-Sections ⊥ x-axis.

Cross-Sections are Semi-Circle.



Radius of Semi-Circle

$$V=\int_0^4\boxed{\text{Area of}}\,dx$$
Cross-Section

$$=\int_0^4\frac{\pi y^2}{2}\,dx$$

Area of Cross-Section $=\frac{\pi r^2}{2}=\frac{\pi(y)^2}{2}$

$$=\int_0^4\frac{\pi}{2}x^2\,dx$$

$=\boxed{}$

Find f_{ave} for $f(x) = \frac{3}{(x+1)^2}$ on $[1,6]$

$$f_{ave} = \frac{1}{b-a}\int_a^b f(x)\,dx$$

$$f_{ave} = \frac{1}{6-1}\int_1^6 \frac{3}{(x+1)^2}\,dx$$

$x=1 \to u=2$

$x=6 \to u=7$

$u = x+1 \qquad du = dx$

$$= \frac{3}{5}\int_1^6 \frac{1}{(x+1)^2}\,dx = \frac{3}{5}\int_2^7 \frac{1}{u^2}\,du = \boxed{}$$

Find c in [2,5] such that

$f(c) = f_{ave}$ in $[2,5]$ for $f(x) = (x-3)^2$

$f(c) = (c-3)^2$

$$f_{ave} = \frac{1}{5-2}\int_2^5 (x-3)^2\,dx$$

$u = x-3$

$du = dx$

$x=2 \to u=-1$

$x=5 \quad u=2$

$$= \frac{1}{3}\int_{-1}^2 u^2\,du$$

So

$f(c) = f_{ave}$

$(c-3)^2 = 1$

Solve for c

$$= \frac{1}{3}\cdot \frac{u^3}{3}\Big|_{-1}^{2} = \frac{1}{9}\left[2^3 - (-1)^3\right]$$

$$= \frac{9}{9} = 1$$

Find all numbers b such that $f_{ave} = 3$ in $[0, b]$ for $f(x) = 2 + 6x - 3x^2$.

$$f_{ave} = \frac{1}{b-0}\int_0^b (2+6x-3x^2)\,dx = 3$$

$$\frac{1}{b}\left[2x + 3x^2 - x^3\right]\Big|_0^b = 3$$

$$\frac{1}{b}\left[2b + 3b^2 - b^3\right] = 3$$

$$2b + 3b^2 - b^3 = 3b$$

$$b^3 - 3b^2 + b = 0$$

$$b(b^2 - 3b + 1) = 0$$

$\rightarrow b = 0$

$b \neq 0$

$b^2 - 3b + 1 = 0$

$\uparrow$

Solve for b.